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Assessment of maintenance strategies for railway vehicles using Petri-nets

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Abstract

The density of railway traffic has been steadily increasing over past years and decades. The developments have implicated a growing need for efficient operation and maintenance of railway rolling stock systems. Also the increased operation of articulated trains has induced new challenges on maintenance organization and planning.

Selecting optimal maintenance strategies for each component does not only influence the availability of the railway vehicles but also the operational performance and the profitability of the operator. Suitable tools to analyse, compare and optimize different maintenance strategies are therefore required.

Petri nets are such a mathematical tool that and have been applied for maintenance modeling and simulations of different applications. Several types of Petri nets with different properties have been introduced. One of the recently proposed extensions of Petri nets are the Abridged Petri Nets (APN) which fulfill the specific requirements of railway rolling stock maintenance.

In this paper, we propose the application of APN in combination with the Monte-Carlo simulation for railway rolling stock maintenance evaluation. In a first step, the applicability of the APN approach was demonstrated on a theoretical case study comprising a condition based maintenance strategy for a system. In a second case study, several real application case studies were modeled and compared based on the processes and real application field data of three railway vehicle components.

The tool can be further extended by pre-defining selected strategies that be easily implemented within an overall decision support system.

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Keywords: maintenance strategy; railway vehicles; petri-nets; abridged petri nets;

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1. Introduction

The density of railway traffic has been steadily increasing over past years and decades. Some links within the railway networks are even operated close to their capacity limits. Concurrently, also the complexity of the railway rolling stock and the infrastructure systems, and the operational impact of their failures have also risen sharply in the recent years. These developments have implicated a growing need for efficient operation and maintenance of railway rolling stock systems.

Also the increased operation of articulated trains has induced new challenges on maintenance organization and planning. One of the approaches to cope with these challenges has been the introduction of modular maintenance. Particularly, condition-based maintenance strategies have been gaining attention and need to be evaluated before a shift from classical maintenance strategies can be implemented.

An adequate maintenance strategy should always consider the operational parameters of the system and meet the strategic goals of the company in terms of reliability, availability and cost. Selecting optimal maintenance strategies for each component does not only influence the availability of the railway vehicles but also the operational performance and the profitability of the operator. Suitable tools to analyse, compare and optimize different maintenance strategies are therefore required that fulfill the essential preconditions of railway vehicles, are flexible and the results of which are easy to interpret.

Several approaches have been introduced to evaluate different maintenance strategies (Wang et al. 2007; Madlener et al. 2015; Huynh et al. 2012). These include Bayesian Networks (Bouillaut et al. 2013), fuzzy linguistics (MECHEFSKE & WANG 2003) and Monte-Carlo simulations (Marseguerra et al. 2002).

Petri nets have been increasingly applied to model and simulate maintenance strategies (Clavareau & Labeau 2009; Volovoi 2004; Hosseini et al. 2000). Petri nets present an approach to mathematically describe processes based on basic set theory. Several extensions have been introduced to Petri nets, such as timed Petri nets, stochastic Petri nets (Volovoi 2004), colored Petri nets and dualistic Petri nets (Dawis et al. 2001). Particularly these extensions have made Petri nets more attractive for the applications to maintenance engineering and planning. They enable the modelling of the different maintenance process steps and are able to model the failure and degradation behavior based on defined stochastic behavior.

Recently, a new compact type of Petri nets has been introduced: the Abridged Petri nets (Volovoi 2013). They are similar to stochastic Petri nets. However, they provide a more compact and intuitive model representation and enable a more intuitive understanding of the modeled processes. If the Abriged Petri nets are combined with Monte-Carlo simulation, they also provide a good tool for evaluating the maintenance strategies quantitavely.

In this paper, we propose to apply the Abridged Petri nets to maintenance strategy evaluation of railway rolling stock components and demonstrate how their specific requirements and boundary conditions can be implemented in Petri net models to enable a suitable and easy to use decision support tool.

2. Current maintenance challenges in the railway industry

A regular maintenance is essential to preserve the railway vehicles and their components in the operational state operable and remedy faults and failures. Omitted or inadequate maintenance activities can cause train delays, cancellations, hazardous situations and even fatal accidents. These adverse events affect the competitiveness and profitability of the railway operators and are therefore of high importance.

The complexity of maintenance processes has been increasing. This is due to several reasons. Articulated trains have been progressively replacing train compositions comprising locomotive and passenger coaches. This results in more complex requirements on maintenance planning. Additionally, purely mechanical systems are progressively being replaced by complex mechatronic devices and systems that combine mechanical, electronic and information technology, with very different degradation and failure characteristics. Selecting an optimal maintenance strategy and evaluating its impact on the reliability, availability and also the life cycle costs of the system is therefore indispensable. The evaluation of maintenance strategies requires a modelling of the impact of the possible alternative strategies on system performance and also on the life cycle costs. However, the increased complexity of the systems and also the increased requirements on the performance of the systems in terms of their reliability, availability and safety have amplified the complexity of evaluating the maintenance strategies.

One of the ways to cope with the increased complexity and also the requirement for a high availability of railway vehicles is the modular maintenance. The modular maintenance concept implies that the vehicles are designed in such a way that the line replaceable units can be replaced as an entire module and can afterwards be revised or repaired independently in the workshop in order thereby increasing the availability of the vehicle.

Despite the increasing use of condition monitoring devices and consequently also the availability of the data on the system condition, the majority of maintenance tasks in the railway industry, is dominated by the traditional maintenance concept: either corrective or periodically scheduled maintenance.

3. Introduction to Petri nets

3.1. Basic principles of Petri nets

Petri nets are a model for describing discrete, distributed and dynamic systems both mathematically and graphically. The approach was originally developed by Carl Adam Petri for modelling chemical processes (Petri 1962). They are not only able to model deterministic processes, but also non-deterministic and concurrent processes. Therefore, Petri nets can be interpreted as a generalization of finite automata.

In the recent years several extensions and further developments have been introduced to the Petri nets, such as the ability to model stochastic processes, dynamic behaviour and hierarchical models, which made them applicable to many different applications. There are several extensions to the initially introduced, the so-called low-level Petri nets with a simple structure. They are used in computer science, logistics, business processes, manufacturing processes and basically all areas with discrete and distributed processes or systems. Petri nets are on the one hand easy to use and interpret and can therefore also be used by non-specialists. On the other hand, they are exact mathematical tools and therefore allow a precise and detailed process analysis.

The definition and information about the structure of Petri nets refer to the low-level Place-Transition Petri net (PTN). It is one of the simplest and most popular Petri nets. The PTN may be considered as a bipartite, directed graph with two types of nodes. The nodes can be either places or transitions, whereby the finite sets of places and transitions are always disjoint.

Places are passive components. They may contain tokens and are often represented as a circle. A place represents for example the state of an object on a conveyor belt. Transitions are active components, they can transport or modify tokens and are often shown as a rectangle. A transition is for example a machining process on the conveyor belt with inputs and outputs.

The nodes are connected with directed edges. An edge always connects a place with a transition or vice versa, and is shown as an arrow (Fig. 1).

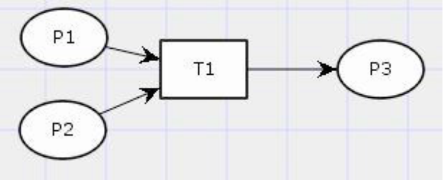

Fig. 1. A Place-Transition Petri-Net example

The dynamics of the network can be visualized using the tokens. They represent the components or for example vehicles in the system. Places can have any number of tokens, which can for example be used to represent the capacity of a system.

3.2. Selection of a suitable type of Petri nets

There are a lot of different high-level Petri nets available: colored Petri nets, timed Petri nets, stochastic Petri nets, object-oriented Petri nets and hierarchical Petri nets. They all have various properties, but not every Petri net is suitable for modelling the maintenance processes.

The following list comprises the requirements on the mathematical model requires for modelling real maintenance processes:

- Several tokens in a place: representing several components in the maintenance model.
- Different (colored) tokens: The system may include different components that may have different failure mechanisms and failure rates. The coloured tokens and transition rules are intended to implement these features in a Petri net.
- Priorities need to be defined for certain tokens. The transitions can fire differently and therefore assign a priority to the token colors depending on the component type.
- Different probability distributions in the transitions: The transitions can fire deterministically or stochastically. The life times of the systems and components are modelled by exponential or Weibull distributions, while the repair times are typically modelled by the lognormal distribution.
- Capacity and resource limitations in workshops or spare part store places
- Ability to model highly complex systems and their interaction (which is enabled by hierarchical networks)
- Ability to analyse and monitor the dynamic behaviour analytically: ability to integrate a Monte Carlo simulator
- All of there requirements are met by the Abridged Petri Net (Volovoi 2015).

3.3. APN Petri Net

The Abridged Petri nets were first introduced by A. Volovoi (Volovoi 2013). This kind of Petri nets is very similar to the stochastic Petri net. Both of them can represent stochastic transition rates, which is one of the basic requiremens for modeling maintenance processes.

The main difference between the stochastic and the Abridged Petri Net is their representation. The tranditional Petri nets have two types of nodes: places and transitions. Contrary to the traditional Petri Nets, the edges between the places in the Abridged Petri nets represent also the transitions. This means that there is now only one type of nodes and an explicit representation of transitions is omitted. The graph now comprises only circles (places) and edges (transitions).

ABN enables the modelling of various properties of tokens, similarly as in the colored Petri nets. Additionally, also the represenation of a hierarchical structure (networks in networks) is also supported. Abridged Petri nets are therefore a very compact type of high-level Petri nets and combine the advantages of stochastic, colored and hierarchical nets. All of these properties are required to model the degradation behavior. Additionally, it can be enriched with a Monte Carlo simulator.

3.4. Lifetime distributions

The lifetime of each component is stochastic and can be modelled by a defined probability distribution. For the lifetime distributions of technical components, particularly the exponential and the Weibull distributions provide a good representation of the actual failure, aging and degradation behaviour. This property can be directly linked to the points in time of the stochastic firing events within the Petri nets. Thereby, different properties of the degradation behaviour can be modelled.

The parameters that need to be specified for the Weibull distribution are the characteristic lifetime and the shape parameter.

3.5. Monte Carlo Simulation

The Monte Carlo simulation is an approach that relies on random repeated sampling and can be applied to obtain approximations of mathematical or statistical problems for which other exact approaches are either difficult or impossible (Dubi 1998). The simulation in the following way: random parameters are generated based on the probability distribution of the input parameters, numerical calculations are performed on these parameters, the steps are repeated for the defined number of repetitions and the results are aggregated (Dubi 1998). For the considered case studies, 100,000 runs are performed which enables a good approximation of the real processes.

In dynamic Petri nets, the sequence of transitions in each step is always non-deterministic. Therefore, the obtained results differ for each run. In stochastic Petri nets, even the individual transitions can have stochastic transition rates, such as for example for the lifetime distributions.

Therefore, Monte Carlo (MC) simulations are required to evaluate the maintenance strategies modeled with Petri nets. The MC can be used to simulate the Petri nets and to evaluate the relevant performance parameters, such as the availability of the systems, the distribution of the repair times, or its statistical indicators, such as the mean, the system failure rate and the number of the performed repairs.

Further to evaluating the system performance, also the associated costs, both direct and indirect need to be evaluated to enable a complete analysis of the consequences of a chosen maintenance strategy. To enable a fair comparison of the different maintenance strategies, the costs over a defined lifetime period of a railway rolling stock system are evaluated. Additionally, also the average costs per year can be computed.

The difference between evaluating the performance parameters of the system and the cost evaluations is that additional monetary values for different transitions are introduced, such as the cost per inspection, repair, and indirect costs. For example, the number of transitions at the maintenance workshop indicates how many times the system was repaired. By introducing a cost for each time the system enters this state, the total repair costs can be calculated. If the cost depends on the time the system spent in a defined state, such costs can also be computed. The costs of all possible states are then summed up. The resulting sum is the total life cycle cost within a given time interval. It is important to note that the investment costs were not considered since this did influence the comparison of the maintenance strategies.

4. Implementation of a condition based maintenance strategy based on theoretical data

The condition based maintenance relies on the fact that the condition of the system and its evolution in time can be either monitored continuously or in defined intervals. If the condition based maintenance is implemented successfully, the consumption of the lifetime of a component can be significantly increased compared to scheduled preventive maintenance, while preventing operational disruptions and maintaining a high level of system reliability and availability. While the benefits of the condition based maintenance can be significant, also the costs can be substantial. The costs include for example the implementation costs of condition monitoring systems, such as sensors or measuring devices, costs of inspections and resulting loss of system availability, but also costs of false alarms, for example due to evolving environmental and operating conditions, imprecise measurements or a high noise level in the measurements. Particularly the false alarms may result in unnecessary replacement of the components and can become quite costly.

To evaluate the feasibility of the selected methodology and the developed model, in the first step a general condition based maintenance model is developed following the principles introduced in (Volovoi 2015). For the initial evaluation of the suitability of the approach synthetic data is used.

To model the degradation behavior, two different states are defined. The system starts in the fully operational state. It is assumed that the system enters the condition “initial failure” after the failure condition becomes measurable. However, depending on the measuring devices and the inspection intervals, the initial failure condition can also remain undetected. If the initial failure condition is detected in time, the component can be replaced preemptively, if not, the component fails, causes operational disruptions and also higher maintenance costs. Fig. 2 represents the possible scenarios. Please note, the monitoring of the component’s condition in this model is performed continuously and not periodically. This increases the probability of the preemptive replacement.

It is assumed that the initial failure condition can only be detected in the last 10% of the lifetime of the component. The model implies therefore, that 90% of the lifetime is spent in the operational state, which represents the transition t_1 between the operational state and the initial failure condition. A probability is assigned to detect the initial failure condition or to miss it. In the model applied in this paper, a detection probability of 95% is assumed. The two outgoing transitions from the state "initial failure condition" are in accordance with the detection probability of the initial failure. The transition t_2 represents the case when the condition monitoring system detects the initial failure in time and changes the color attribution of the tokens from “*0*” to “*1*”. If the initial fault remains undetected, the transition t_3 is activated and the color of the token remains “*0”*. This probabilistic process has been modeled with the help of two transitions with exponential distributions. The rate of the two transition rates is proportional to the desired detection probabilities. In this model the detection probability is 95%, the "lifetime" of the transition t_2 is therefore $9.5 * 10^6$ and t_3 is $5 * 10^5$. They fire very quickly and represent this way proportionately the detection probability.

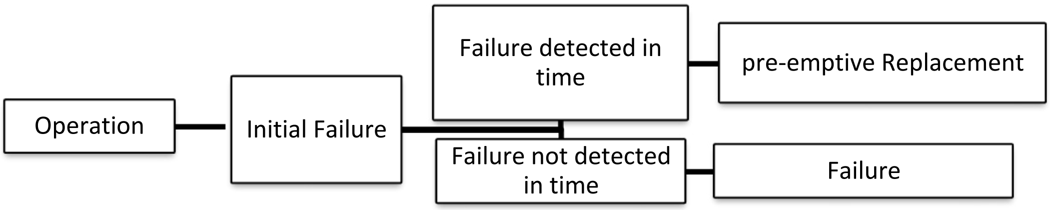

Fig. 2 Flowchart of the theoretical condition based maintenance strategy

If the initial failure condition is detected, the component is pre-emptively replaced or repaired. The repair time is represented by the transition t_4. Please note, that the path of the pre-emptive replacement is only possible for the tokens with the color “*1*”, which represent those components the failure condition of which was detected on time. The transition t_7 changes the color of the token back to 0. If the measurement system does not trigger an alarm, which occurs in 5% of the cases, the initial failure condition remains undetected; the color of the token remains *“0”* and the component fails after a while. Fig. 3 presents the Petri net model.

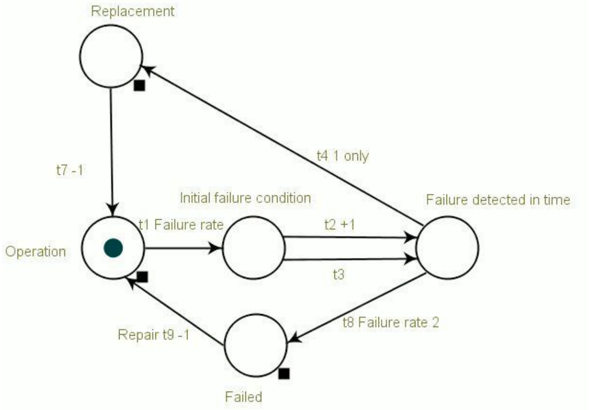

Fig. 3 APN Petri net model of the theoretical condition based maintenance strategy

Table 1 provides an overview on the input data and the results of the MC simulation of 100 years. The observation time period was chosen to be so large to enable sufficiently frequent observation of the events of interest. The initial failure condition could be detected 34 times on average during the observation period and the component could be replaced pre-emptively. The component failed 1.7 times on average, because the fault condition was not triggered in time. This small number corrective maintenance assignment is due to the well-functioning condition monitoring system. This enable average yearly maintenance costs of 1,870 CHF.

Table 1 Input Data of the simulation

Lifetime of the component t_1	1.9 years; Weibull distributed with shape parameter 6
Lifetime after the detection trigger (t_8)	0.1 year, Weibull distributed with shape parameter 6
Detection probability of the initial failure condition	95%
Duration of pre-emptive replacement	0.01 year (approx. 3.5 days); deterministic
Duration of the corrective repair	0.01 year (approx. 3.5 days); deterministic
Cost of pre-emptive replacement	5,000 CHF
Cost of corrective maintenance	10,000 CHF

5. Implementation of a condition based maintenance strategy based on real data

The model developed for the synthetic dataset is now adapted to real industrial conditions with three components from two different rolling stock fleets. These components have been chosen because the condition based maintenance strategy is already in use for these components in the industry. This chapter contains models of real maintenance strategies of the train operator. Three components have been selected from two different train types with two lifetime values (30 and 40 years). It's about two air conditioners and a front coupling head. These Petri net models and the simulation results can be used as a support tool in strategic decisions. The simulations were performed on the models with real data, but in this paper, both the input data, and the simulation results were falsified because of the non-disclosure agreement. The input data are shown in the Table 2. Failure costs contain the replacement costs also. The indirect costs are costs of delays, lost hours and dissatisfaction of passengers. The results of the cost calculations are rounded to hundred francs.

Table 2 Input data for the simulation of the condition based maintenance strategies

	Air condition system Train1	Air condition system Train2	Front coupling head Train1
Lifetime of the vehicle	40 years	30 years	40 years
Component's lifetime	2.2 years	26 years	2.6 years
Inspection intervall [days]	365	365	150
Inspection costs [CHF]	326	240	1,470
Failure costs (Failure not in time detected) [CHF]	5,480	4,300	11'070
Repair time [Hours]	5	13.5	13.5
Indirect Costs [CHF]	5,600	5,600	27,200

The detailed flow chart of the model can be seen in the previous chapter, the only difference in the model is an internal clock with the transitions T_{10} and $T1_1$. T_{11} represents the revision interval (annual or semi-annual). T10 represents how long the "Inspection due" condition remains active. The duration of the transition greatly affects how many inspection actions are carried out, because the places "Inspection due" and "Failure detected in time" should contain both at the same time a token so that the inspection is performed. Fig. 4 shows the flow chart and Fig. 5 the model of this maintenance strategy.

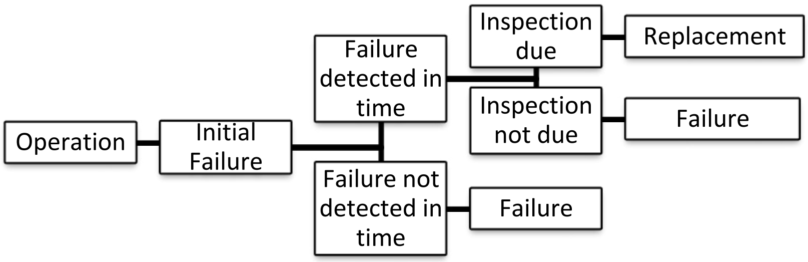

Fig 4 Flowchart of the condition based maintenance model with inspections

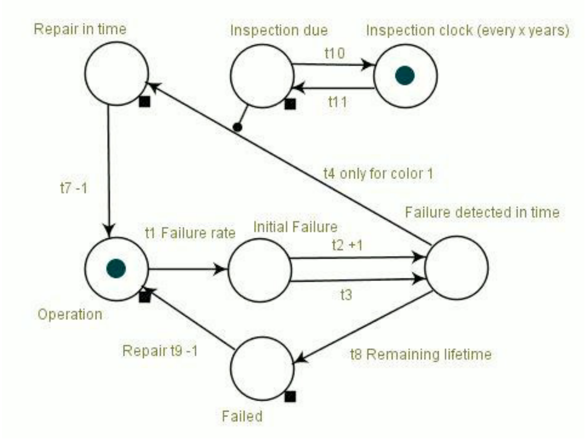

Fig 5 APN Petri net model with inspections

The indirect costs and the failure costs are due in case of failure and the cost of inspection in case of an inspection. It does not matter how many times an inspection was performed successfully, the cost of inspections should always be paid. The total costs include the inspection-, failure and indirect costs. The line "Cost per year" means the average annual cost of the maintenance strategy.

The table 3 shows, for example, that the air conditioner of the train 1 is inspected 40 times in 40 years and failed 16 times. From the 40 inspections were 3.1 successfully which means that the components have been replaced in time in the initial failure state and therefore they did not fail.

The two components of the vehicle 1 have a shorter lifetime than the air conditioner of the train 2, therefore makes this strategy more sense for these and a part of the failures can be prevented.

It was also investigated how high would be the maintenance costs without regular inspections. It is shown on the table 3 in the last two lines. However, there are no inspection costs in this case, but the number of failures is increased. If the costs per year are compared, it can be noticed that the strategy with the annual inspections is cheaper. The reason for this is that the cheaper inspections may prevent some expensive failures.

Table 3 Results of the simulation for models 1 and 2

		Air condition system Train1	Air condition system Train2	Front coupling head Train1
Model 1with inspection				
Failure				
	Number of failures	16	0	11
	Cost [CHF]	177,300	0	421,000
Inspection				
	Number of inspections	40	30	80
	Cost [CHF]	13,000	7,200	117,600
	Number of interventions	3.1	0.2	5
	Total cost in 40/30 years [CHF]	190,300	7,200	538,600
	Annual cost [CHF]	4,800	200	13’500
Model 2 without inspection				
	Number of failures	19.2	0.9	16.2
	Annual cost [CHF]	5,300	200	15,500

5.1. Further development of the model

Two different other strategy variants (models 3 and 4) were also simulated with this condition based model. In the first experiment, the frequency of inspections was doubled. For example, look at the air conditioner of the train 1. The costs of the inspections have doubled, but the number of failures is decreased by 30% and therefore the cost per year are lower by 20%. (See Table 4), this strategy would be a good choice at least looking from the cost side. It must be also taken into consideration that the more frequent inspections have a negative impact on the availability of the vehicle and require more staff.

TABLE 4 Results of the simulation of the first variation (model 3)

Model 3		Air conditioner Train1	Air conditioner Train2	Front coupling head Train1
Failure				
	Number of failures	11.7	0	3.8
	Cost [CHF]	129,600	0	145,400
Inspection				
	Number of Inspections	80	60	160
	Cost [CHF]	26,100	14,400	235,200
	Total cost in 40/30 years [CHF]	155,700	14,400	380,600
	Annual cost [CHF]	3,900	500	9,500

In the second strategy variant the difference from the previous models is that the remaining lifetime after the detection of the fault condition (transition T8) is not 10% of the total lifetime, but always fix 4 months, irrespective of the lifetime. Among the components of the vehicle 1, it means an increase of the remaining lifetime, because it was previously 10% of the total lifetime, about 2 months (0.2 years). If there is more time after the trigger of the failure condition, there are a higher chance to detect the failure during an inspection; Therefore, the number of failures is less than in the original case.

The advantage of this strategy variant is that the annual costs are cheaper than in the previous versions and this number of inspections must not be increased. (See Table 5) This means that the availability of the system can remain high. The increase in the remaining lifetime or an earlier failure detection can be achieved in practice with a better measurement method or the procurement of a new measurement instrument. My suggestion is that the failure should be noticed as early as possible, even if it requires new technologies or devices.

Table 5 Results of the simulation of the second variation (model 4)

Model 4		Air condition system Train1	Air condition system Train2	Front coupling head Train1
Failure				
	Number of failures	12	0.7	5.8
	Cost [CHF]	133,000	0	222,000
Inspection				
	Number of inspections	40	30	80
	Cost [CHF]	13,000	6,900	117,600
	Total cost in 40/30 years [CHF]	146,000	14,100	339,600
	Annual cost [CHF]	3,700	500	8,500

6. Conclusion

There were 4 different condition based maintenance models investigated in this paper. The basis of the comparison was the maintenance cost, therefore, the annual maintenance costs of each strategy were gathered together in tabular form. (Shown in the Table 6)

Table 6 Summary table

Modell Nr.	Air condition Train1	Air condition Train2	Front coupling head Train1
		Annual cost [CHF]	
1	4‘800	200	13‘500
2	5‘300	200	15‘500
3	3‘900	500	9‘500
4	3‘700	500	8‘500

The costs of the condition based models are mainly influenced by the frequency of inspections. If there aren't any inspections performed, the costs are going to be higher due to the higher number of failures (Model 2). But if the inspections are more frequent, the failures are detected with higher chance and the component fails less often and the cost is decreasing (Model 3). The 4th model is the best solution where the remaining lifetime after the error detection is the longest, therefore fail even less components. But here is needed a better measurement system or technology and that is not taken into account in the costs yet.

The aim of this research was to develop a tool based on Petri nets that can be used in decision support for selecting maintenance strategies. The developed support tool consists of several Petri net models and the associated cost calculation tables. The methodology developed was tested on a simple theoretical system and then applied to maintenance processes of the SBB

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