Efficient timetable stability analysis using a graph contraction procedure

Bütikofer Stephan, Fontana Dominik, Steiner Albert, Wüst Raimond

Zurich University of Applied Sciences, School of Engineering, Institute of Data Analysis and Process Design Winterthur, Switzerland

Abstract

Train density on the Swiss rail network has increased significantly in recent years. This demands much more from the stability of the system, especially in combination with single-track corridors. For railroad companies, it is therefore becoming increasingly important to optimize the transport network for stability in order to be able to offer the demanded service as reliably as possible on the existing infrastructure. An approach to numerical stability evaluation of timed discrete event systems has been developed to support planners in testing the timetable for operational stability. In this approach, the traffic system under consideration is modeled as a network with all relevant timetable events and links. The system modelled in this way can be examined for its behavior in the event of a possible disruption using methods from max-plus algebra. The typical computation time of an evaluation procedure takes more than 65 minutes for a signifcant partition of the line network. This is far too high for integration into a practical optimization procedure.
In this paper we present a contraction procedure added to the existing evaluation framework, and thus reduce the computation time by more than 90% without compromising the result quality.

Keywords: timetable stability, max-plus algebra, event activity network, graph contraction

1 Introduction

In 2019, 200 million train kilometres was covered on the Swiss railway network by passenger traffic. This represents an increase of 43% compared to the year 2000 [1]. The shortening of train headway times associated with this rapid growth places much higher system stability requirements. On largely single-lane networks, such as that of the Rhaetian Railway (RhB), there is also the challenge of over-proportionally increasing intersection events, which affect not only subsequent but also oncoming train runs.

With the increasing density of services, it is becoming more important for railway companies to have suitable planning tools at hand to optimize the systems stability. Additionally, the Swiss Integrated Fixed-Interval Timetable provides strict periodicity-conditions onto new timetable offers, e.g., the new Rhaetian 'Retica30+' timetable [2]. This again places higher system stability requirements.

For this, the introduction of a holistic numerical stability analysis as part of the planning process is recommended. In this way, those parts of a timetable that are particularly susceptible to delays, or are even a chain reaction of delays, could already be identified and defused during the planning process.

An approach for numerically evaluating timetable stability is by applying the so-called max-plus algebra to timed discrete event systems (TDES, see [5]). The traffic system under consideration is set up as an event-activity network (EAN) with all relevant nodes and links, representing timetable arrival and departure events with the linking process times. The processes considered are operational trip times, headways, stopping times, transfer times, crossing times, transit times and turning times.

The system model created in this way can be examined for its stability in case of a disruption. The typical algebraic calculation time of an evaluation process for a moderately sized part network of the RhB is still too high (65 minutes) for an integration into a practical optimisation procedure. A practical application of this method is described in [3].

In this paper we present a method for contracting the network graph before applying the max-plus stability analysis, to speed up the evaluation process without compromising the quality of the results. The acceleration method is applied to a timetable scenario of a representative part of the RhB network (see Figure 1, right).

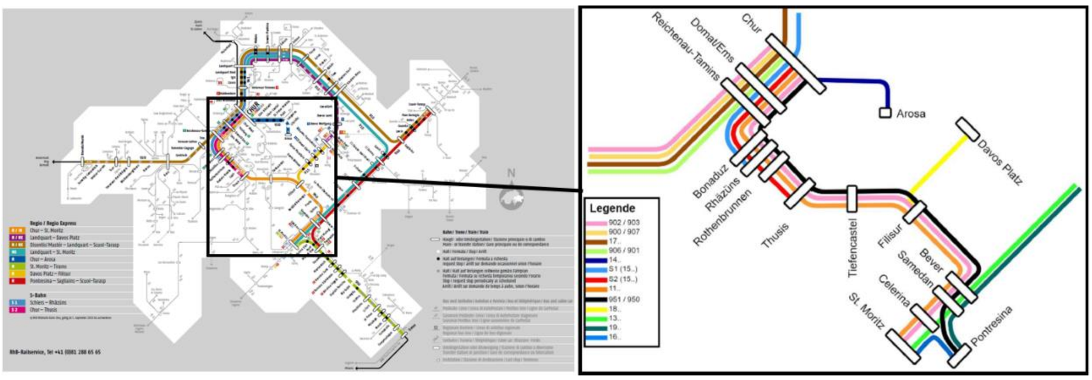

Figure 1: The RhB railway network. Left: entire network, right: test scenario

The results of the accelerated procedure are compared with those of the original procedure based on the uncontracted EAN. The reduced EAN size, the calculation times and the calculated indicators will be analysed, evaluated, and discussed.

2 Methods

Max-plus algebra

An advantage that results from the formulation of a traffic system as a TDES is the possibility of describing and analyzing the system using the max-plus algebra. This approach makes it possible to formulate the recursive calculations per event as a linear system of equations, which among other aspects enables an efficient generation of the desired indicators, see [4], [5] and [6]. All these indicators of timetable stability can be derived from the critical cycle.

Critical cycle and performance indicators

In a periodic timetable, departure and arrival events are repeated periodically, usually hourly. A periodic sequence of events and process times (see introduction) is called a cycle. This cycle begins and ends with the same event at the same geographical location on the rail network. Several train runs of different lines can occur within a cycle. The critical cycle in a system is the one that has the largest cycle mean or the least buffer time to the timetable periodicity [6] and [7].

The largest cycle mean is the length of time in which all cycles of the system can be carried out at least once. Its meaning for the evaluated timetable stability indicators CDS (cumulative delay sensitivity) and CDI (cumulative delay impact) of each timetable event can be derived from the critical cycle of a timetable. The formal descriptions are presented in detail in Wüst et al. [7] and Steiner [8].

Graph contraction

In the context of stability analysis, graph contraction is based on the idea that first- or second-degree events, i.e., events with at most one incoming and/or one outgoing activity do not influence the system stability and thus its upstream or downstream activitie(s) can be removed and numerically re-linked in a suitable way. This is due to the assumption that the remaining entries of the recovery matrix should provide the same result after the described contraction.

This approach is proposed by Goerigk and Liebchen in [9] and [10] for enabling a more efficient optimization of PESP problems.

In the case of a periodic EAN, as it is considered here, there are no first-degree nodes (events), since each graph released for analysis must consist of a single component, i.e., exclusively consist of events that are accessible from each other event. This implicates that only second-degree events that have two neighboring activities must be contracted. This means that the corresponding events are removed, and their respective activities are linked together. At this new link, the respective process times are added. This contraction step is illustrated based on an example EAN.

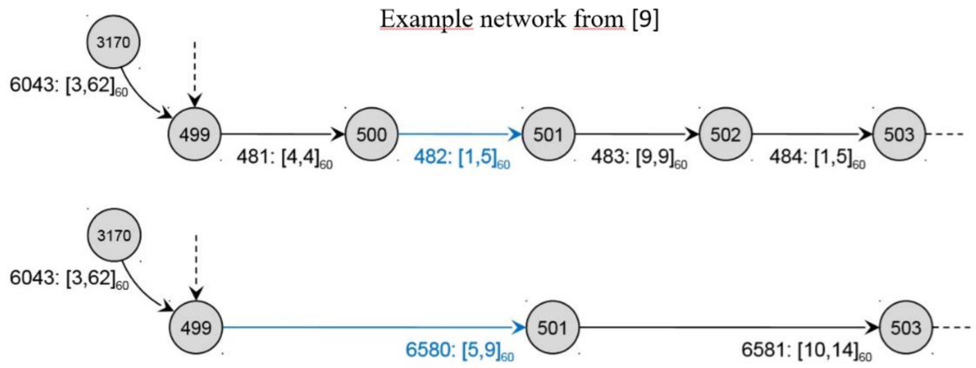

Figure 2: Example network from [9] before and after contraction of the second-degree event, where the process times (time intervals in brackets) of the original links were added to that of the new link

3 Results

The results obtained using the contraction method of [9] address two aspects, the performance gain and the quality of the obtained results.

Performance
The performance gain consists mainly in the reduced computation time for generating the stability indicators of Table 2 that is achieved by reducing the number of graph elements. For all calculations we used a workstation with a six-core Intel Core i7 8700K, a maximum clock frequency of 4.70 GHz and 16 GB of RAM. All calculations were carried out several times. Mean values were calculated after checking for statistical outliers.

Graph contraction and computation time
In Figure 3 is illustrated, that the contraction method reduces the number of events (factor 2.5) and activities (factor of almost 3). This leads to the reduction of the computing time for generating the indicators from around 4000 seconds to about 100 seconds. 50 seconds were needed for data preprocessing (contraction plus formating max-plus input data).

.

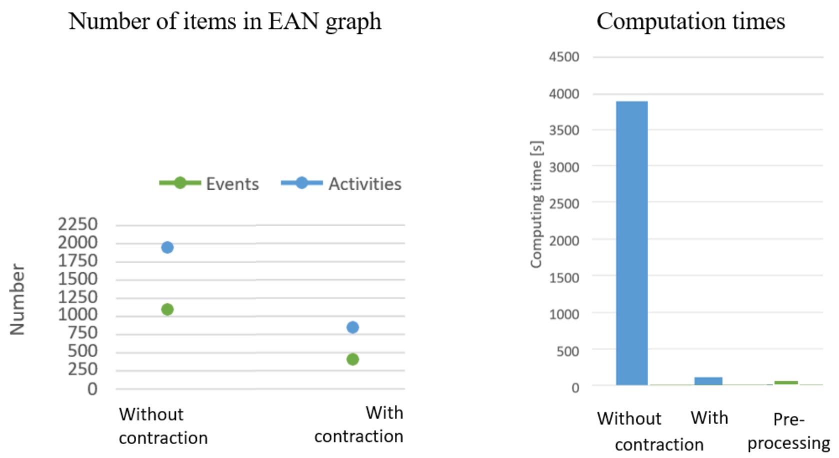

Figure 3: Left side: Number elements in EAN of each scenario. Right side: Computation time for generating stability indicators without and with contraction (blue bars) and for contracting the EAN graph (green bar)

Quality of results

The quality of the results is given in terms of the absolute and relative deviation of the stability measures for the contracted compared to those of the original network and thus aims at setting the performance gain obtained by the reduction process in relation to the results without the use of this method.

In Table 2 the critical cycles of both versions are compared to each other. The length of the critical cycles represents an absolute quality measure. The length of the critical cycle is identical in both versions.

Critical cycle	Without contraction	With contraction
Critical cycle identical?	-	yes
Eigenvalue λ [min]	51.31	51,31

Table 2: Analysis of the critical cycles of all process combinations in both scenarios.

The second quality aspect concerns the cumulative delay impact of each event. To achieve better comparability between the different scenarios, the CDIs were weighted by the number of existing events in the system. The delay impact is a relative quality measure for comparing the delay sensitivity of different events of a given line, rather than evaluating absolute values. Figure 4 shows the plots of the CDIs of train run 1125 Chur - St. Moritz. The top plot represents the calculation without, the lower one with graph contraction. It can be seen how the CDIs of this train run grow in both plots between Tiefencastel (TICA) and Filisur (FILI). This is because in the critical cycle a potential delay is translated from another train run to the train run 1125 in Thusis (THS). These plots illustrate the good relative agreement between the two methods.

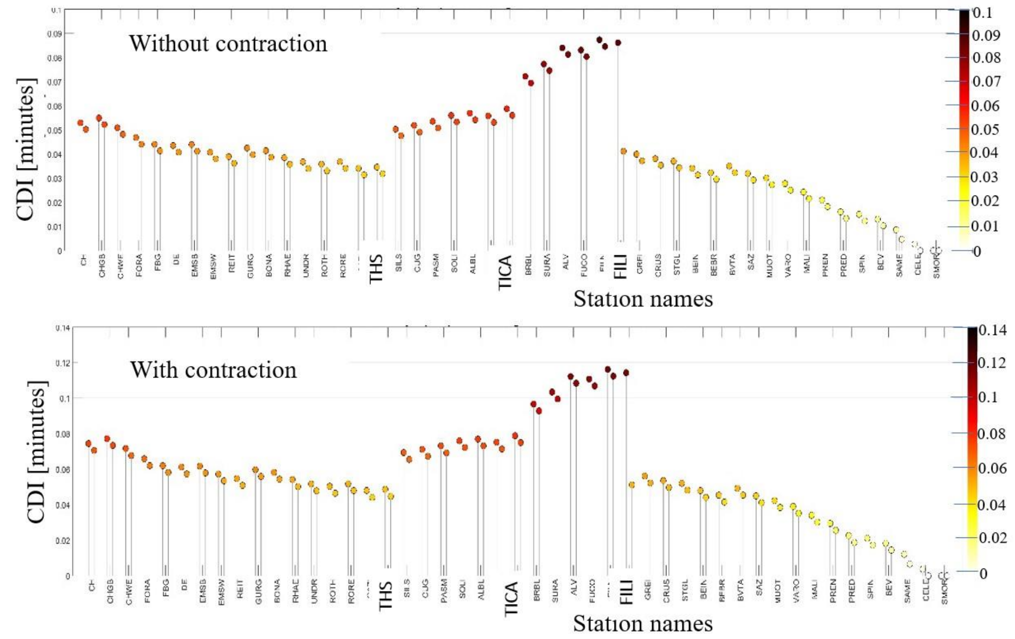

Figure 4: Cumulative delay impact (DCI) of train run 1125 Chur - St. Moritz in comparison. Departure and arrival event per station. Top: Without graph contraction, Bottom: With graph contraction.

4 Conclusions and Contributions

The analyses presented in this paper show that by applying the graph contraction method, a computing time saving of 96% can be achieved on a timetable scenario of the magnitude of the RhB Albula corridor.

In large scenarios, in which data preprocessing plays a rather subordinate role, the underlying relationship between scenario size and computing time results from the number of elements in a matrix of size N x N, where N is the number of events in the system.

N determines the computation time of the path matrix A+ (see recovery time in table 1) and scales with N3.

It has been shown that the procedure omits information on timetable dependencies that are not relevant for the stability of the system.

Besides the critical cycles the values of the cumulative delay impact of each timetable event have a strong practical relevance. Timetable events with a high CDI have strong influence on all other timetable events. Timetable planners should try to consider measures to increase corresponding buffer times and thus decrease the CDI values. In operations these events require special attention to ensure that they occur at the planned times and thus to prevent disruptions.

If critical events are depending on the reliable functioning of track infrastructure, like for instance certain switches or safety elements, special attention must be paid to preventive maintenance.

Figure 4 illustrates that these stability indicators have differently scaled absolute values depending on whether the contraction method is applied or not. However, there is a very good agreement between both methods what concerns the relative values.

Since in practice it is more important to identify the critical events rather than knowing the absolute values of the overall delay impact, applying the contraction method satisfies this requirement.
This makes graph contraction the recommended method for efficient timetable stability analyses. The effectiveness of the process depends largely on the so-called connectivity of the network. The method works better with lower connectivity, as such networks are richer in second-degree events on which the process is based.
On the other hand, the procedure can be used for the semi- or fully automatic optimization of new service concepts, annual timetables or replacement timetables during maintenance work.
In addition, this method can be useful for applications in which a rough but fast overview of the status of large systems is required.

References

[1] Bundesamt für Statistik (2021), «Leistungen im Personenverkehr». [Online] https://www.bfs.admin.ch/bfs/de/home/statistiken/mobilitaet-verkehr/personenverkehr/leistungen.html#642897470. [Accessed on 25.05.2021].

[2] Rhaetian Railway (2012), «Ein Meilenstein - für Graubünden und die RhB». [Online] https://www.rhb.ch/de/medien/medienmitteilungen/details/ein-meilenstein-fuer-graubuenden-und-die-rhb. [Accessed on 25.05.2021]

[3] R. M. Wüst, S. Bütikofer, S. Ess, C. Gomez, A. Steiner, M. Laumanns, J. Szabo, (2019), "Improvement of maintenance timetable stability based on iteratively assigning event flexibility in FPESP". Planning, Simulation and Optimisation Approaches, 8th International Conference on Railway Operations Modelling and Analysis 2019, Norrköping.

[4] Chr. Pöppe (1995), „Fahrplan-Algebra", Spektrum der Wissenschaft Vol. 11/1995, p. 20, Spektrum der Wissenschaft Verlagsgesellschaft mbH. [Online] https://www.spektrum.de/magazin/fahrplan-algebra/822683

[5] R. M. P. Goverde (2010), „A delay propagation algorithm for large-scale railway traffic networks," Transportation Research Part C: Emerging Technologies Department of Transport and Planning, Vol. 18, Issue 3, June 2010, P.269-287.

[6] R. M. P. Goverde (2007), „Railway timetable stability analysis using max-plus system theory", Transportation Research Part B: Methodological, Vol. 41, Issue 2, February 2007, P.179-201.

[7] R. M. Wüst, C. Gomez, S. Ess, A. Steiner und S. Bütikofer, (2019) „Development of a prototype for the automated generation of timetable scenarios specified by the transport service intention", https://digitalcollection.zhaw.ch/bitstream/11475/18564/5/SBBLab_final_report_update_20190628_v3.pdf

[8] A. Steiner, (2018) „Einführung in die Max-Plus-Algebra und Berechnung von Stabilitäts- und Performancemassen für den Schienenverkehr," Zürcher Hochschule für angewandte Wissenschaften, Winterthur.

[9] M. Goerigk and Chr. Liebchen, (2017) „An Improved Algorithm for the Periodic Timetabling Problem," Department of Management Science, Lancaster University; Technical University of Applied Sciences Wildau, Lancaster, Wildau, 2017. [Online] https://drops.dagstuhl.de/opus/volltexte/2017/7892/ [Accessed on 25.05.2021]

[10] Chr. Liebchen, (2018) „On the Benefit of Preprocessing and Heuristics for Periodic Timetabling," Technische Hochschule Wildau, Ingenieur- und Naturwissenschaften, Verkehrsbetriebsführung, Wildau, 2018. [Online] https://ideas.repec.org/h/spr/oprchp/978-3-319-89920-6_94.html [Accessed on 25.05.2021]