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PID parameter tables after ITAE to control overshooting systems found with AI algorithm

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***Abstract*—Embedded controllers are used in many applications. It often happens that these do not offer any possibilities to set the controller parameters, because they are capsulated to outside. The closed-loop systems are then stable, but the systems are often weakly damped and their step responses exhibit overshoot. In the following, a method is described, how such embedded controlled systems can be controlled via a superimposed feedback loop using PID controllers in such a way that the system deviation of the step response is minimized according to the ITAE method. A method from artificial intelligence, hill climbing, is used for this purpose. For the control of such systems, the paper essentially provides PID controller parameters, which are usable for many weakly damped systems. The parameters are verified in a practical application with a speed control of a motor with inertial mass.**

Keywords—PID control, ITAE, artificial intelligence

I. Introduction and Related Research

In the figure 1, there is shown an embedded controlled system with a superimposed control. The behavior of such embedded controlled systems is often as a black box, and then it is not possible to change the parameters or intervene in the system in any way. Many of these systems are not optimally adjusted and exhibit an overshooting behavior as a 2nd order system. Therefore, a PID controller is applied as a superimposed control. PID controllers are still most common used in such applications.

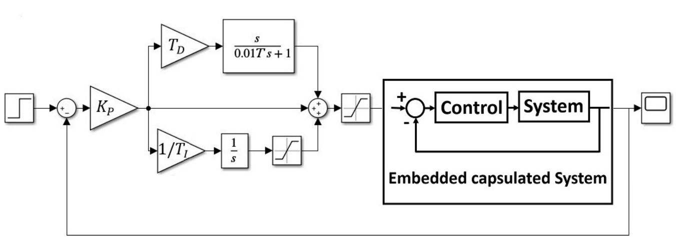

Fig. 1. Superimposed control of an embedded capsulated system.

It is therefore necessary that on the one hand the actual output value or the signal of the sensor can be picked up at the output. On the other hand, it must also be possible to feed the setpoint into the system.

There are many methods and approaches to find suitable controller parameters. The first method for this was that of Ziegler-Nichols [1], but there are also several others known, such as that of Chien, Hrones and Reswick [2]. However, they only apply to time-delayed systems and not to overshooting systems. Most of these approaches are heuristic and the controllers found need to be further optimized. Today, however, also various other methods are in use to optimize controllers.

In recent years, some methods in the topic of artificial intelligence, as examples particle swarm optimization (PSO) and also some others [3-9] have also been used for the optimization of controllers. Further related methods [10-16] were also used for optimization. In addition, there are existing many other optimization methods for the tuning of control parameters [17-20]. Good converging algorithms for the control with H∞ criterions were investigated in [3 – 5]. And also Mechatronic systems were regulated using such methods [6, 7]; these methods were used to control motors which are connected directly to wheels [6]. In addition, a new algorithm with a new particle type for the PSO algorithm was introduced, this was in the following also successfully used to control a non-linear system [8]. PSO was also used for the tuning of PID controllers for an electronic commutated DC Motor and also servo motors [9]. There were also other methods developed. In [11] a new self-tuning PID temperature regulator was developed using a neural network. In [12] a genetic algorithm was developed, which optimizes the parameters of PID control to do a position regulation of the electrohydraulic actuators.

But the PSO methods can just find a local minimum. There, these parameters for Kp, Ti and Td are treated as particles in three dimensions. These are then calculated in iterations, based of random starting values from one iteration to another, using the three-dimensional vector for velocities. For these particles, the PSO algorithms use several components with different weights: these are the vectors from last iteration, and a velocity

vector, which points into the direction of best criteria achieved using the specific particle, furthermore also a velocity vector, which points into the direction of the best criteria over all the particles. There is also the component of the social behavior of the particles, which influences the calculation.
Here, the related method ‘hill-climbing’ [20] is introduced for the minimization of the ITAE criterion of PID controllers. It is similar to PSO. But the difference is, that that all particles are calculated serially with the algorithm, but not parallel. Here, also the social behavior is missing, there is no connection of the different particles. As result of this, much more different local minima of the criteria are found and this will increase the probability that smaller ITAE criterions will be found. At the end, the smallest one of all minima is then accepted as result parameter set. This method is described below.

II. Identification of a 2ND order overshooting system

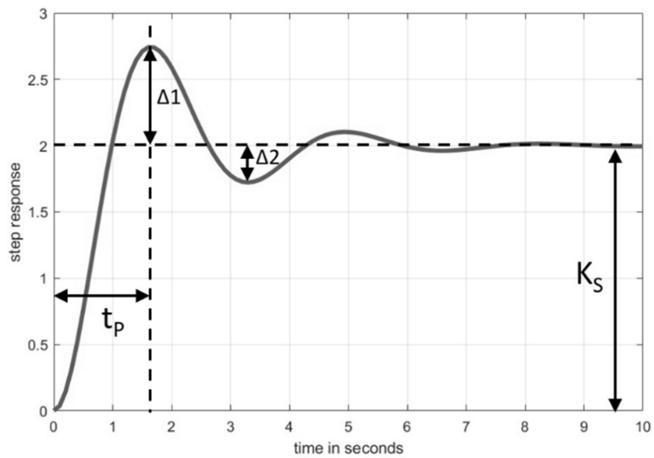

Fig. 2. The step response of a damped 2nd order system.

However, although many systems are stable, they exhibit system-related overshooting. In addition to the encapsulated systems shown in figure 1, there are also other weakly damped systems, for example spring-mass systems, which have to be controlled in the application of an active vibration damping. Figure 1 shows such a step response.
Simple rules can be defined using figure 2 in order to read the parameters of the transfer function from a given step response of an overshooting system. The transfer function G(s) becomes as described below. Such systems are very common in practice, but will be described here for further use in the paper’s tables.

$$G(s) = \frac{K_s}{T^2 \cdot s^2 + 2 \cdot D \cdot T \cdot s + 1} \qquad (1)$$

First the overshoot Δ1 is found. This is specified in relation to the stationary final value. From the graph in figure 2 one reads: Δ1 = 0.37. From this, the relationship between the first overshoot Δ1 and the system damping D can be calculated [21].

$$\Delta 1 = \exp\left(-\frac{D \cdot \pi}{\sqrt{1-D^2}}\right) \qquad D = -\frac{\ln(\Delta 1)}{\sqrt{\pi^2 + (\ln(\Delta 1))^2}} \qquad (2)$$

With the given system according to figure 2, this results in D = 0.3. In practice, the damping can also be read from the graph in figure 3 [22].

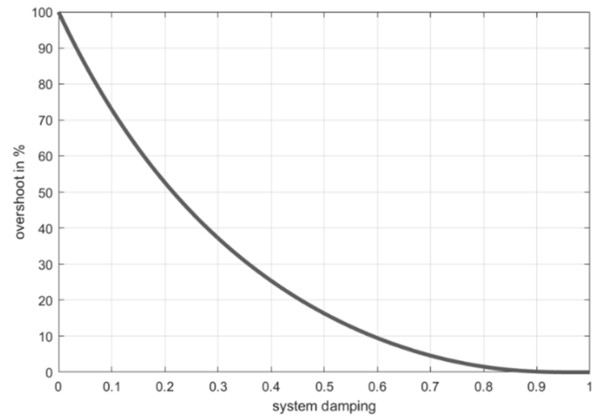

Fig. 3. Overshoot of a 2nd order system in function of the system damping.

Furthermore, the time constant T must also be identified. For this purpose, the time tp is measured according to figure 2. This is the time between the begin of the step and reaching the maximum overshoot. The time constant T is directly related to tp.

$$T = \frac{t_p}{\pi} \qquad (3)$$

For the example of the step response according to figure 2, there is T = 0.5s. The graphic represents a response to a unit step. Since the static end value is equal to 2, there is Ks = 2:1 = 2 in this example. The system with the parameters according to the figure 2 is therefore complete identified.

$$G(s) = \frac{K_s}{T^2 \cdot s^2 + 2 \cdot D \cdot T \cdot s + 1} = \frac{2}{0.25 \cdot s^2 + 0.3 \cdot s + 1} \qquad (4)$$

The stable second-order systems discussed in this document have a damping D between 0 and 1. Their step responses show an overshoot behavior as shown in figure 2. Other second order systems with step responses without overshoot behavior have a damping D of > 1.0. Their control is dealt with in [18].

III. Hill climbing method to calculate the controller parameters after the minimized ITAE criterion

After the system was identified, it can be represented with the transfer function. All of the following considerations and calculations are based on the block diagram of figure 4. It represents the closed loop circuit of the controlled system. The system described above is shown there on the right. Depending on the damping factor D, it overshoots more or less. The PID controller is shown on the left. The integrator is designed with an output limitation. When using the optimal parameters of the controller calculated in the following, this is never active. However, these control output limitations are always built into practical designs. So that the PID controller can be implemented, a filter must also be added to the differential component. Its time constant is chosen here to be one hundred times faster than the time constant T of the second-order system. The controller output limitation between the controller and the second order system is also present in all practical systems. This will be taken into account in the parameter sets, which are calculated below.

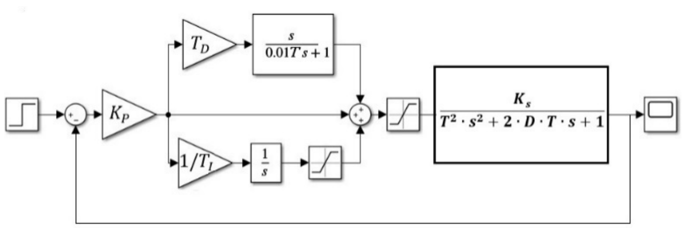

Fig. 4. Closed loop control loop, as it is used for all calculations.

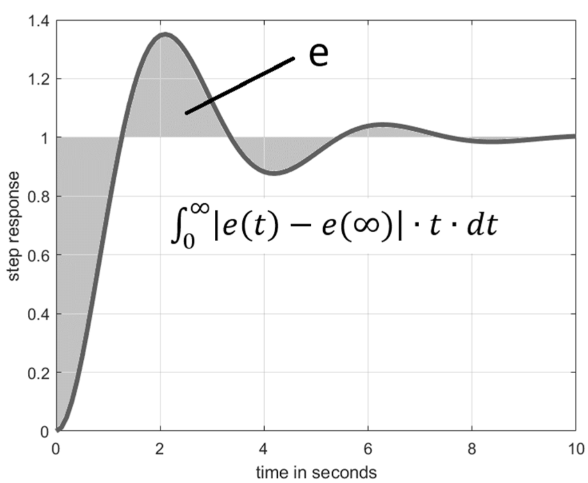

Fig. 5. The ITAE criterion in the time domain.

In figure 5, there the criterion is drawn. This is the area of the deviation of a step response of the system from unit step. And It is also weighted by time, it is therefore multiplied by the time. Thus, the error has a greater influence when the time progresses.
In theory, it is not difficult to calculate these controller parameters according to the minimal ITAE criterion. You only have to calculate all the possible parameter sets of P, I, D (Kp, Ti, Td) for the second order systems, using different damping masses D and controller output limitations.

However, when calculating the optimal parameters for the PID controller, there the challenge arises that one has to calculate a large number of simulations. The three parameters span a three-dimensional space. There are also two further dimensions for the calculation of the parameter sets with different controller output limitations and also the system dampings D. Thus the problem could be solved in polynomial time, but the complexity is x5 and it requires too much calculating capacity. Therefore, the calculation was made with the 'hill-climbing', that can find at least local minima.
There is carried out a multiplication of a small change of new iteration with a random value (0, +1 and -1). This iteration is a minimal change in the controller parameters, for Kp and Ti and Td. It is then added to the parameters. After that, the ITAE criterion is recalculated. If the new value of the criterion is smaller, the new parameter sets are used for the next calculation. If not, then the old parameters are used.

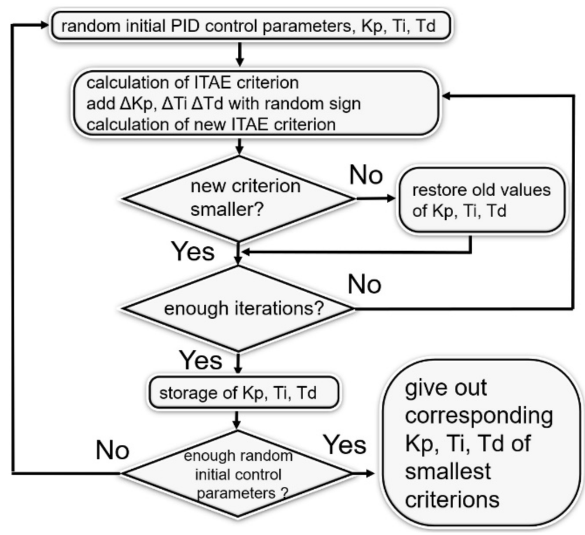

Fig. 6. Flow chart of the hill climbing method.

However, this method only finds local minima. Therefore, many different random tuples of start values of the controller parameters were used in the parameter search. After calculation, many solutions of the converged minimum ITAE criteria agree with each other. Therefore, one can assume with reasonably good certainty that the parameters found are actually the PID parameters Kp, Ti and Td, which correspond to the absolute minimum of the ITAE criterion. In any case, they are at least very close to them.

IV. PID PARAMETER TABLE FOR OPTIMAL ITAE

In table 1, the parameter sets calculated in this way for different system dampings D and different controller output limitations

are shown. It is noteworthy that the table scales with the static gain Ks and the time constant T. This makes the table usable for all subcritical damped systems of 2nd order. This table is the core of this publication.
The meaning of +/− (2,3,5,10) is the controller output limitation. This is also part of the table and influences the parameter sets. Furthermore, the maximum values of the table are limited to 10.

TABLE I. PID PARAMETERS FOR MINIMAL ITAE CRITERION

ITAE	(controller limitation value – controller output before the step) divided by (controller output for stationary end value – controller output before the step)			
D	**+/- 2**	**+/- 3**	**+/- 5**	**+/- 10**
1.0 0% overshoot	Kp = 10/Ks Ti/T = 9.6 Td/T = 0.3	Kp = 10/Ks Ti/T = 7.3 Td/T = 0.3	Kp = 9.6/Ks Ti/T = 5.4 Td/T = 0.3	Kp = 9.8/Ks Ti/T = 4.7 Td/T = 0.3
0.9 0.2% overshoot	Kp = 9/Ks Ti/T = 8.4 Td/T = 0.35	Kp = 9.7/Ks Ti/T = 7.1 Td/T = 0.35	Kp = 10/Ks Ti/T = 5.4 Td/T = 0.3	Kp = 9.9/Ks Ti/T = 4.5 Td/T = 0.3
0.8 1.5% overshoot	Kp = 9.7/Ks Ti/T = 8.7 Td/T = 0.35	Kp = 9.9/Ks Ti/T = 7.0 Td/T = 0.35	Kp = 9.8/Ks Ti/T = 5.5 Td/T = 0.35	Kp = 9.9/Ks Ti/T = 4.8 Td/T = 0.35
0.7 4.6% overshoot	Kp = 10/Ks Ti/T = 8.6 Td/T = 0.35	Kp = 10/Ks Ti/T = 6.8 Td/T = 0.35	Kp = 10/Ks Ti/T = 5.4 Td/T = 0.35	Kp = 9.9/Ks Ti/T = 4.6 Td/T = 0.35
0.6 9.5% overshoot	Kp = 9.8/Ks Ti/T = 8.3 Td/T = 0.4	Kp = 10/Ks Ti/T = 6.9 Td/T = 0.4	Kp = 10/Ks Ti/T = 5.2 Td/T = 0.35	Kp = 9.9/Ks Ti/T = 4.9 Td/T = 0.4
0.5 16% overshoot	Kp = 9.9/Ks Ti/T = 8.1 Td/T = 0.4	Kp = 9.8/Ks Ti/T = 6.5 Td/T = 0.4	Kp = 9.8/Ks Ti/T = 5.3 Td/T = 0.4	Kp = 9.9/Ks Ti/T = 4.7 Td/T = 0.4
0.4 25% overshoot	Kp = 9.7/Ks Ti/T = 7.6 Td/T = 0.4	Kp = 10/Ks Ti/T = 6.4 Td/T = 0.4	Kp = 10/Ks Ti/T = 5.2 Td/T = 0.4	Kp = 9.9/Ks Ti/T = 4.5 Td/T = 0.4
0.3 37% overshoot	Kp = 9.4/Ks Ti/T = 7.3 Td/T = 0.45	Kp = 9.7/Ks Ti/T = 6.3 Td/T = 0.45	Kp = 9.9/Ks Ti/T = 5.4 Td/T = 0.45	Kp = 9.9/Ks Ti/T = 4.8 Td/T = 0.45
0.2 53% overshoot	Kp = 9.7/Ks Ti/T = 7.3 Td/T = 0.45	Kp = 9.9/Ks Ti/T = 6.2 Td/T = 0.45	Kp = 9.9/Ks Ti/T = 5.2 Td/T = 0.45	Kp = 9.9/Ks Ti/T = 4.6 Td/T = 0.45
0.1 73% overshoot	Kp = 9.9/Ks Ti/T = 7.5 Td/T = 0.5	Kp = 9.8/Ks Ti/T = 6.3 Td/T = 0.5	Kp = 10/Ks Ti/T = 5.5 Td/T = 0.5	Kp = 9.9/Ks Ti/T = 4.9 Td/T = 0.5
0.0 100% overshoot	Kp = 10/Ks Ti/T = 7.3 Td/T = 0.5	Kp = 10/Ks Ti/T = 6.2 Td/T = 0.5	Kp = 10/Ks Ti/T = 5.3 Td/T = 0.5	Kp = 9.9/Ks Ti/T = 4.7 Td/T = 0.5

V. CONTROL OF A MECHATRONIC SYSTEM AS APPLICATION

As an example, a speed control of a motor is realized. In figure 7, a very common system of a motor with a load is showed. The load is represented by a rotating wheel with a mass moment of inertia J. The brushless DC or DC motor is driven by an amplifier. The speed is measured and calculated via an encoder.

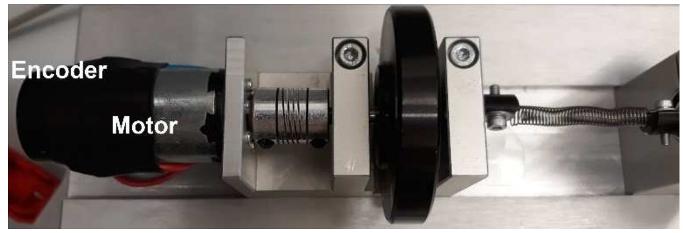

Fig. 7. (Brushless) DC motor with a load, with a mass moment of inertia J.

It is now assumed that the rotational speed of this system is controlled with an encapsulated embedded system. This means that its controller parameters are not accessible from the outside and cannot be changed. The step response of the system controlled in this way is shown in figure 8.

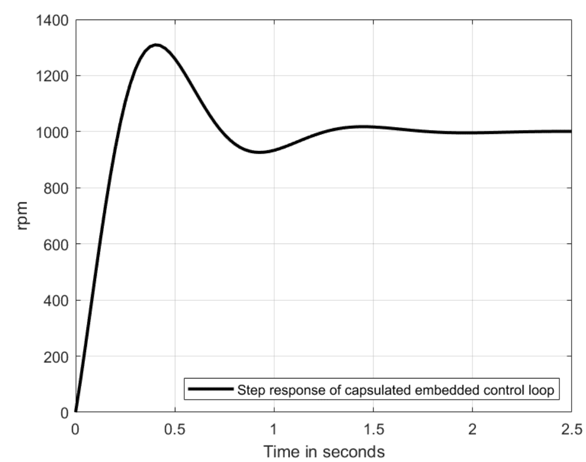

Fig. 8. Step response of the capsulated embedded control loop.

The system regulated in this way is already stable. The response to a jump to 1000 rpm is shown here. One would now like to regulate this system using the parameter table found in the last chapter in such a way that the ITAE criterion is minimized. For this purpose, a superimposed control according to the block diagram of figure 9 is used. In order to determine the correct control parameters according to the parameter table, the system with the step response from figure 8 will be identified. This is done according to the procedure outlined above. This results in a time constant T = 0.13 and a system damping D = 0.33 and Ks = 1. The transfer function G(s) is thus calculated as

$$G(s) = \frac{K_S}{T^2 \cdot s^2 + 2 \cdot D \cdot T \cdot s + 1} = \frac{1}{0.0169 \cdot s^2 + 0.0858 \cdot s + 1} \quad (5)$$

With this system, the conversion of the encoder corresponds to 200 rpm/V. Therefore A setpoint step from 0 to 1000 rpm corresponds to a step from 0 to 5V. The motor amplifier is supplied with +/-15V. Thus it can give a maximum of 15V voltage to the motor. The control output limitation factor is therefore ± 15V / 5V = ± 3. The parameters Kp, Ti and Td are calculated according to the table values, with the approximate value D = 0.3.

$$K_P = \frac{9.7}{K_S} = 9.7 \quad T_i = 6.3 \cdot T = 0.819 \quad T_d = 0.45 \cdot T = 0.0585 \tag{6}$$

This results in the block diagram according to figure 9.

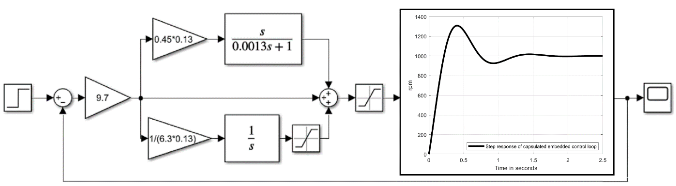

Fig. 9. Closed loop of the controlled system.

Finally, for this controlled system with the PID parameters read from the table for the minimized ITAE criterion, the very nice step response of the new closed loop system according to figure 10 results. The table values perform the system very well.

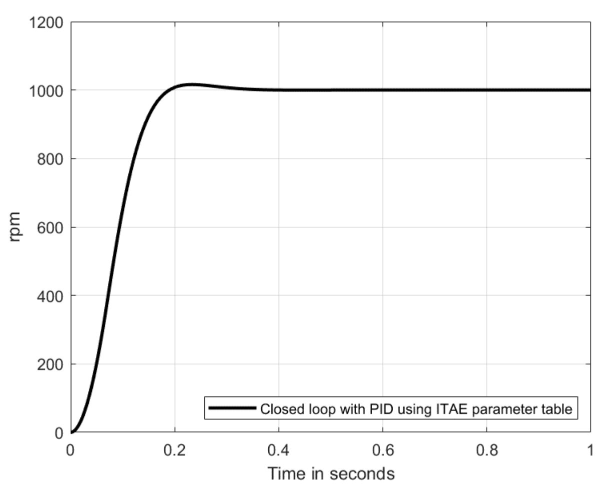

Fig. 10. Step response of the system controlled according to the parameters of the minimized ITAE criterion.

VI. Discussion and Outlook

The step response of this system and also other ones discussed is very fine. This is clear in that sense, because the parameters of the PID controllers in the table were calculated before and then selected as the best ones. This example can show very well, that the PID controllers are not only usable for the control systems that do not exhibit any overshoot. Very good regulation parameters can also be found for the systems with overshoot, using the table, which was calculated with the 'hill climbing' method discussed here. These also result in good step response behavior according to the minimum ITAE criterion.

And particularly, also the output limitations were used for the parameter calculation. These are always part of the real existing systems and influence the PID parameters. The calculated parameters are also dependent of system parameters T and D, and the amplification Ks. It can also be shown that the poles of the new closed loop overall systems also scale with it. Therefore, the controller parameters shown in the table can be used very generally. Therefore these parameter sets are usable for a wide range of such weakly damped systems, which show an overshoot behavior that are out in practice, in just using the table which is shown here.

This publication may contribute to find adequate controller parameters for the systems to be controlled in practice, which show such step responses.

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